CHAPTER

4 Mathematics of Finance

WHAT IS TIME VALUE OF MONEY AND WHY IT IS SO IMPORTANT?

Time Value of Money (TVM) is an important concept that money in hand today is worth more than a money promised in the future. Basically, it is a fundamental financial concept that recognizes the worth of money over time. It emphasizes the principle that a sum of money available today is worth more than the same amount of money in the future. This is primarily because money has the potential to earn returns or interest when invested or utilized immediately.

WHY IS INTEREST PAID?

Interest compensates for the time value of money, recognizing that receiving money earlier is generally more beneficial than receiving it later. By paying interest, the borrower compensates the lender for the temporary use of their funds.

- **Opportunity Cost:** When individuals or organizations lend money, they forgo potential alternative uses of those funds. The interest paid serves as compensation for the opportunity cost of not investing or using the money elsewhere.
- **Inflation:** Inflation erodes the purchasing power of money over time. Lenders charge interest to account for the expected decrease in the value of money due to inflation. This ensures that the lender's real purchasing power is maintained or increased.
- **Liquidity Preference:** Lenders may require interest as compensation for the lack of immediate access to their funds. By lending money, they forgo the ability to use that money for their own immediate needs or expenses.
- **Risk Factor:** Interest also accounts for the risk associated with lending money. Lenders assess the creditworthiness of borrowers and charge interest rates based on the perceived risk of default. Higher-risk borrowers are charged higher interest rates to compensate for the increased probability of non-repayment.

  These factors collectively determine the interest rate charged on loans, bonds, or other financial instruments, allowing lenders to earn a return on their capital while borrowers gain access to funds for various purposes.

DEFINITIONS

Interest: Fee paid for the use of another party's money. To the borrower it is the cost of renting money, to the lender the income from lending it.

E.g.: The principal amount invested by Anita is ₹10,000. After 1 year, she received the accumulated amount or balance, including both the principal and the interest, as ₹10,500.

Thus, the difference between ₹10,500 and ₹10,000 which is ₹500 is the interest she earned in a year.

Principal: The total amount of money borrowed (or invested), not including any interest.

E.g.: Sarah wants to buy a new laptop and decides to take a loan of ₹30,000 from a finance company. The ₹30,000 is the principal amount she borrows, not including any interest.

Rate of Interest: The interest rate (rate of interest) is the rate at which interest is charged on the principal amount which is landed or borrowed. Rate of interest is usually expressed as percentages.

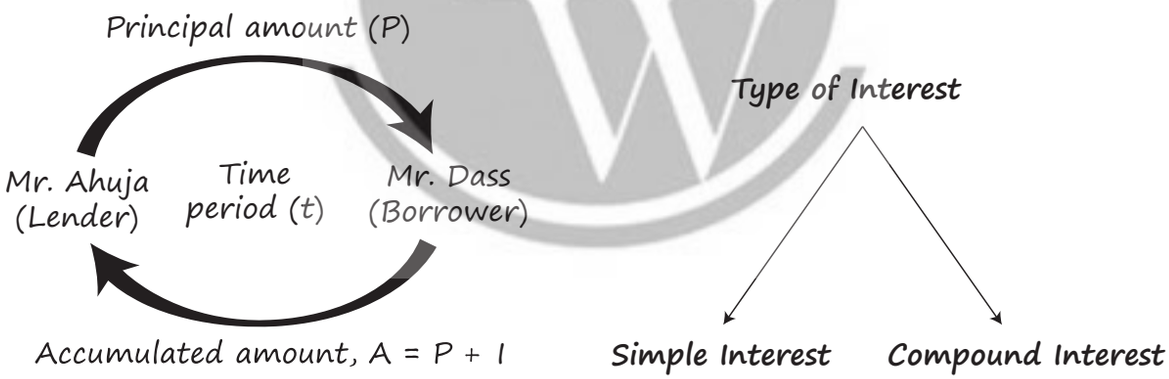
E.g.: Mark wants to invest his savings in a fixed deposit with a bank. The bank offers an annual interest rate of 6% on fixed deposits. The 6% is the rate of interest at which Mark's money will earn interest over time.

Time Period: The time period refers to the duration for which the principal amount is borrowed or invested, and during which interest is charged or earned. It is the length of time over which the financial transaction takes place.

E.g.: If John borrows ₹50,000 from a bank at an annual interest rate of 8% for a time period of 2 years, then the time period in this case is 2 years.

Accumulated amount (or Balance): The amount which you receive after the due period including the principal and the interest in called as accumulated amount.

E.g.: Ankita invested ₹20,000 in a savings account that earns an annual interest rate of 4%. After three years, she checked her account balance and found it to be ₹22,408. The ₹22,408 is the accumulated amount or balance, including the principal and the interest earned over the three-year period.



SIMPLE INTEREST

- Simple interest is the way when we take interest at a simple rate on the principal amount over a period of time.
- Simple interest increases (or decreases) if Principal amount increases (or decreases) or if interest rate increases (or decreases) or if the time period increases (or decreases).
- It means Simple interest(I) is directly proportional to Principal Amount (P), Interest rate (R%), Time period (T), Therefore,

$$S.I. = \frac{P \times R \times T}{100}$$

Example 1. S.I. on ₹3,500 for 3 years at 12% per annum is **(ICAI)**

(a) ₹1,200 (b) ₹1,260 (c) ₹2,260 (d) None of these

Sol. (b) Given, Principal (P) = ₹3,500

Time (T) = 3 years

Rate (R) = 12% p.a.

We know that,

$$S.I. = \frac{P \times R \times T}{100}$$

$$S.I. = \frac{3500 \times 12 \times 3}{100} = 1260$$

Therefore, the simple interest on ₹3,500 for 3 years at 12% per annum is ₹1,260.

Calculator trick: I = P × T × R% = 3500 × 3 × 12% = 1,260

Hence, the correct option is (b).

Example 2. Sania deposited ₹50,000 in a bank for two years with the interest rate of 5.5% p.a. How much interest would she earn?

(a) ₹2,750 (b) ₹5,500 (c) ₹55,500 (d) None of these

Sol. (b) Given, Amount deposited in the bank (P) = ₹50,000

Rate of interest (R) = 5.5% p.a.

Time (T) = 2 years

We know that,

$$S.I. = \frac{P \times R \times T}{100}$$

$$= \frac{50,000 \times 5.5 \times 2}{100}$$

$$= 500 \times 5.5 \times 2 = 5,500$$

Calculator trick: I = 50,000 × 2 × 5.5% = 5500

Therefore, she earned ₹5,500 as interest.

Hence, the correct option is (b).

Example 3. If P = ₹5,000, T = 1 year, I = ₹300, R will be **(ICAI)**

(a) 5% (b) 4% (c) 6% (d) None of these

Sol. (c) Given, P = ₹5,000, T = 1 year, I = ₹300

We know that,

$$\Rightarrow S.I. = \frac{P \times R \times T}{100}$$

$$\Rightarrow 300 = \frac{5,000 \times R \times 1}{100}$$

$$\Rightarrow R = \frac{300}{50}$$

$\Rightarrow R = 6\%$

Therefore, the rate of interest is 6%

Hence, the correct option is (c).

Example 4. How much interest will be earned on ₹500 at 10% simple interest for 5 years and what will be the accumulated amount?

(a) ₹125, ₹625 (b) ₹250, ₹500 (c) ₹250, ₹750 (d) None of these

Sol. (c) Given, Principal (P) = ₹500, Rate of interest (R) = 10% p.a. and Time (T) = 5 years

We know that,

$$S.I. = \frac{P \times R \times T}{100}$$

$$= \frac{500 \times 10 \times 5}{100} = ₹250$$

Thus, the interest earned is ₹250.

Now, accumulated amount (A) = P + I

A = 500 + 250

A = ₹750

Therefore, the interest earned is ₹250 and the accumulated amount is ₹750.

Calculator trick: $I = 500 \times 5 \times 10\% = 250$

A = P + I = 500 + 250 = 750

Hence, the correct option is (c).

Example 5. P = ₹12,000, A = ₹16,500, T = $2\frac{1}{2}$ years. Rate percent per annum simple interest will be **(ICAI)**

(a) 15% (b) 12% (c) 10% (d) None of these

Sol. (a) Given that:

$P = ₹12{,}000$, $A = ₹16{,}500$ and $T = 2\frac{1}{2} = \frac{5}{2}$ years

We know that,

$$I = A - P \text{ and } I = \frac{P \times R \times T}{100}$$

where,

A = Accumulated amount

P = Principal

T = Time in years

R = Rate of interest per annum

Thus, I = 16,500 − 12,000 = ₹4,500

We have, $I = \frac{P \times R \times T}{100}$

$$\Rightarrow 4500 = \frac{12000 \times R \times 5}{2 \times 100}$$

$$\Rightarrow 4500 = 60 \times R \times 5$$

$$\Rightarrow R = \frac{4500}{5 \times 60} = 15\%$$

Therefore, the rate percent per annum is 15%.

'OR'

We know,

$$A = P\left(1 + \frac{RT}{100}\right)$$

$$16500 = 12000\left(1 + \frac{R(2.5)}{(100)}\right)$$

$R = 15\%$

Calculator trick: Steps to follow:

1. 16500 ÷ 12000
2. −1
3. ×100
4. ÷2.5 which gives the required result i.e., 15%.

Hence, the correct option is (a).

Example 6. P = ₹8,500, A = ₹10,200, R = $12\frac{1}{2}$% SI, T will be **(ICAI)**

(a) 1 year 7 months (b) 2 years

(c) $1\frac{1}{2}$ years (d) None of these

Sol. (a) Given that:

P = ₹8,500, A = ₹10,200 and R = 12— % = $\frac{25}{2}$ % years

We know that,

$$I = A - P \text{ and } I = \frac{P \times R \times T}{100}$$

Thus, I = 10,200 − 8,500

$\Rightarrow$ I = ₹1,700

Now, $I = \frac{P \times R \times T}{100}$

$$\Rightarrow 1700 = \frac{8500 \times 25 \times T}{2 \times 100}$$

$$\Rightarrow 1700 = \frac{85 \times 25 \times T}{2}$$

$\Rightarrow 1700 \times 2 = 85 \times 25 \times T$

$\Rightarrow T = \frac{1700 \times 2}{85 \times 25} = 1\frac{3}{5}$ years

Therefore, the time period is $1\frac{3}{5}$ years = 1 year 7 months (approx).

Hence, the correct option is (a).

Example 7. A sum of money amounts to ₹6,200 in 2 years and ₹7,400 in 3 years. The principal and rate of interest are **(ICAI)**

(a) ₹3,800, 31.58% (b) ₹3,000, 20%

(c) ₹3,500, 15% (d) None of these

Sol. (a) **Detailed:**

Given, A_1 = ₹6200 and t_1 = 2 years

A_2 = ₹7400 and t_2 = 3 years

We know that,

$A = P(1 + rt)$

$\Rightarrow A_1 = P(1 + rt_1)$

$\Rightarrow 6200 = P(1 + 2r)$ ...(i)

Also $A_2 = P(1 + rt_2)$

$\Rightarrow 7400 = P(1 + 3r)$ ...(ii)

Now divide equation (i) by (ii), we get

$\Rightarrow \frac{6200}{7400} = \frac{P(1 + 2r)}{P(1 + 3r)}$

$\Rightarrow \frac{31}{37} = \frac{1 + 2r}{1 + 3r}$

By cross multiplication, we get

$\Rightarrow 31 \times (1 + 3r) = 37 \times (1 + 2r)$

$\Rightarrow 31 + 93r = 37 + 74r$

$\Rightarrow 93r - 74r = 37 - 31$

$\Rightarrow 19r = 6$

$\Rightarrow r = \frac{6}{19} = 0.315789 \times 100\% = 31.58\%$

Now, put $r = \frac{6}{19}$ in equation (i) we get

$\Rightarrow 6200 = P\left(1 + 2 \times \frac{6}{19}\right)$

$\Rightarrow 6200 = P\left(1 + \frac{12}{19}\right)$

$\Rightarrow 6200 = P \times \frac{31}{19}$

$\Rightarrow P = \frac{6200 \times 19}{31}$

$\Rightarrow P = ₹3,800$

Therefore, the principal and rate of interest are ₹3800 and 31.57% respectively.

Trick:

If simple interest for 1 year be S.I then

Amount in 2 years = A_2 =P + 2 (S.I)

Amount in 3 years = A_3 =P + 3 (S.I)

Thus $A_3 - A_2$ = S.I

S.I. = 7400 − 6200 = 1200

$\Rightarrow \frac{(P \times R \times 1)}{100} = 1200$

$\Rightarrow$ PR = 120,000

Go by choices:

Option (a): ₹3,800, 31.58%

PR = 3800 × 31.58 = 120004 ≈ 120000

Option (b): ₹3,000, 20%

PR = 3000 × 20 = 60000 ≠ 120000

Option (c): ₹3,500, 15%

PR = 3500 × 15 = 52500 ≠ 120000

Hence, the correct option is (a).

Example 8. A sum of money doubles itself in 10 years. The number of years it would triple itself is **(ICAI)**

(a) 25 years (b) 15 years (c) 20 years (d) None of these

Sol. (c) Given, T = 10 years

Let the Principal be P.

Now, according to the question

Accumulated amount = 2 × Principal

i.e., A = 2P

We know that,

I = A − P, where I = Interest

$\Rightarrow$ I = 2P − P

$\Rightarrow$ I = P

Also, $I = \frac{P \times R \times T}{100}$

$\Rightarrow P = \frac{P \times R \times 10}{100}$

$\Rightarrow$ R = 10%

Now, when A = 3P, then

$I = A - P = 3P - P$

$\Rightarrow I = 2P$

$\therefore I = \frac{P \times R \times T}{100}$

$\Rightarrow 2P = \frac{P \times 10 \times T}{100}$

$\Rightarrow T = 20$ years

Therefore, the sum of money will triple in 20 years.

Hence, the correct option is (c).

Example 9. Find the rate of interest if the amount owned after 3 months is ₹525, borrowed amount being ₹500.

(a) 25% (b) 20% (c) 30% (d) None of these

Sol. (b) Given, Amount (A) = ₹525

Principal (P) = ₹500

Time (T) = 3 months = $\frac{3}{12} = \frac{1}{4}$ years

We know that,

Amount (A) = Principal (P) + Interest (I)

$A = P + \frac{P \times R \times T}{100}$

$\Rightarrow 525 = 500 + \frac{500 \times R \times 1}{4 \times 100}$

$\Rightarrow 525 - 500 = \frac{5 \times R}{4}$

$\Rightarrow 25 = \frac{5R}{4}$

$\Rightarrow 5R = 100$

$\Rightarrow R = 20\%$

Therefore, the required rate of interest is 20%.

Hence, the correct option is (b).

Example 10. Rohit invested ₹50,000 in a bank at the rate of 5.5% p.a. simple interest rate. He received ₹69,250 after the end of term. Find out the period for which the sum was invested by Rohit.

(a) 7 years (b) 5 years (c) 10 years (d) none of these

Sol. (a) Given, Money invested (P) = ₹50,000

Amount received (A) = ₹69,250

Rate of interest = 5.5%

We know that,

Amount (A) = Principal (P) + Interest (I)

$$A = P + \frac{P \times R \times T}{100}$$

$$\Rightarrow 69,250 = 50,000 + \frac{50,000 \times 5.5 \times T}{100}$$

$$\Rightarrow 69,250 = 50,000 \left(1 + \frac{55 \times T}{1000}\right)$$

$$\Rightarrow \frac{69,250}{50,000} = 1 + \frac{55T}{1000}$$

$$\Rightarrow 1.385 - 1 = \frac{55T}{1000}$$

$$\Rightarrow 0.385 = \frac{55T}{1000}$$

$$\Rightarrow T = \frac{385}{55}$$

$$\Rightarrow T = 7$$

Therefore, Rohit made the investment for 7 years.

Hence, the correct option is (a).

Example 11. Kiran took some amount from Suraj for 6 and — years at the rate of 4% p.a. simple interest. And Kiran after that time has to pay ₹31,500 at the end of the term. What amount did Kiran took from Suraj?

(a) ₹25,240.40 (b) ₹30,000 (c) ₹25,000 (d) ₹21,550

Sol. (c) Given, Amount to be paid (A) = ₹31,500, Rate of interest = 4% and Time period (T)

$= 6\ \frac{1}{2} = 6.5$ years

We know that,

Amount (A) = Principal (P) + Interest (I)

$$A = P + \frac{P \times R \times T}{100}$$

$$\Rightarrow A = P \left(1 + \frac{R \times T}{100}\right)$$

$$\Rightarrow 31,500 = P\left(1 + \frac{R \times T}{100}\right)$$

$$\Rightarrow 31,500 = P\left(1 + \frac{4 \times 6.5}{100}\right)$$

$$\Rightarrow 31,500 = P\ (1 + 0.26)$$

$$\Rightarrow 31,500 = P(1.26)$$

$\Rightarrow P = \frac{31,500}{1.26}$

$\Rightarrow P = 25,000$

Therefore, Kiran borrowed ₹25,000 from Suraj.

Hence, the correct option is (c).

Example 12. A farmer borrowed ₹3600 at the rate of 15% simple interest per annum. At the end of 4 years, he cleared this account by paying ₹4000 and a cow. The cost of the cow is **(Dec 2022)**

(a) ₹1,000 (b) ₹1,200 (c) ₹1550 (d) ₹1,760

Sol. (d) Given, P = ₹3600, R = 15% and T = 4 years

We know that,

$S.I. = \frac{P \times R \times T}{100}$

$\Rightarrow S.I. = \frac{3600 \times 15 \times 4}{100}$

$\Rightarrow S.I. = 2160$

Thus, A = P + S.I. = 3600 + 2160 = ₹5760

Since, the farmer cleared the account by paying ₹4000 and a cow.

Thus, the cost of cow = ₹5760 – ₹4000 = ₹1760

Therefore, the cost of a cow is ₹1760 .

Hence, the correct option is (d)

PRACTICE QUESTIONS (PART A)

1. If P = ₹3,600, A = ₹6,400, then Simple interest i.e. I will be

(a) ₹1,800 (b) ₹2800 (c) ₹1600 (d) ₹9000

2. P = ₹12,000; R = 5%; T = 5 years using $I = \frac{PTR}{100}$ then I will be

(a) ₹2,000 (b) ₹3,000

(c) ₹3,700 (d) None of these

3. ₹80,000 is invested to earn a monthly interest of ₹1200 at the rate of _____ p.a. simple interest.

(a) 12% (b) 14% (c) 16% (d) 18%

4. Find the rate of interest if the amount owed after 6 months is ₹1050 borrowed amount being ₹1000.

(a) 9% (b) 10% (c) 11% (d) 12%

5. ₹8,829 is invested into three different sectors in such a way that their amounts at 4% p.a. S.I. after 5 years; 6 and 8 years are equal. Find each part of the sum.

(a) ₹2943 (b) ₹3082.8 (c) ₹2970 (d) ₹3069

6. Raju borrowed ₹10000 from Gopal at the interest rate of 7% for 3 years. How much interest Raju needs to pay in 3 years and how much in total will Gopal receive?
   (a) ₹1,700; ₹11,700 (b) ₹2,000; ₹12,000
   (c) ₹2,100; ₹12,100 (d) ₹2,300; ₹12,300

7. A certain sum of money was put at S.I. for 2.5 years at a certain rate of S.I. p.a. Had it been put at 4% higher rate, it would have fetched ₹500 more. Find the sum of money.
   (a) ₹4000 (b) ₹5000 (c) ₹6000 (d) None of these

8. ₹850 becomes ₹1250 in 5 years at a simple rate of interest. How much would be its amount after 4 years if rate of interest is increased by 5% ?
   (a) ₹1230 (b) ₹1320 (c) ₹1340 (d) None of these

9. A certain sum of money becomes six times at 5% S.I. p.a. At what rate % it will become 12 times.
   (a) 15% (b) 13% (c) 11% (d) None of these

10. How long will it take for a sum to quadruple itself at 10% per year simple interest?
   (a) 13 years (b) 15 years (c) 25 years (d) 30 years

11. A person invests ₹1000 in a savings bank paying 6% simple interest. What is the balance or amount of the savings account after 6 months?
   (a) ₹1030 (b) ₹1060 (c) ₹30 (d) None of these

12. A sum of money invested of compound interest double itself in four years. In how many years it become 32 times of itself at the same rate of compound interest.
   (a) 12 years (b) 16 years (c) 20 years (d) 24 years

Answer Key									
1. (b)	2. (b)	3. (d)	4. (b)	5. (d)	6. (c)	7. (b)	8. (c)	9. (c)	10. (d)
11. (a)	12. (c)								

COMPOUND INTEREST

The interest calculated on the initial principal, which also includes all of the accumulated interest of previous periods of a deposit or loan.

SYSTEM USED BY BANKS

For example: Suppose you deposit ₹1,000,000 in CITI bank for 3 years at 5% p.a. compounded annually. To calculate the Interest, the concept of compound interest is used.

Now, what will be the formula for it?

$$C.I. = P\left(1+\frac{r}{100}\right)^{t} - P$$

$$A = P\left(1+\frac{r}{100}\right)^{t}$$

Example 13. Raju took a loan of ₹10,000 from bank at the compound interest rate of 7% annually for 3 years. How much Raju needs to pay bank now?

(a) ₹12,250.43 (b) ₹22,250.43 (c) ₹2,250.43 (d) None of these

Sol. (a) **Method 1:**

Given, Loan amount (P) = ₹10,000

Time period (t) = 3 years

Rate of interest (r) = 7% p.a.

Since, the interest is compounded annually, thus

$$A = P\left(1 + \frac{r}{100}\right)^t$$

$$= 10000\left(1 + \frac{7}{100}\right)^3$$

$$= 10000(1.07)^3 = 12,250.43$$

Calculation trick:

(i) 1.07 × = =

(ii) Then × 10000

Therefore, Raju needs to pay ₹12,250.43 to the bank.

Method 2:

A = 10000 + 5% + 5% + 5% + 5% = 12250.43

Hence, the correct option is (a).

Example 14. If P = ₹1,000, R = 5% p.a, n = 4; What is Amount and C.I.? **(ICAI)**

(a) ₹1,215.50, ₹215.50 (b) ₹1,125, ₹125

(c) ₹2,115, ₹115 (d) none of these

Sol. (a) Given that :

P = ₹1,000, R = 5% p.a. n = 4

We know that,

$$A = P\left(1 + \frac{r}{100}\right)^t$$

$$\Rightarrow A = 1000\left(1 + \frac{5}{100}\right)^4$$

$$\Rightarrow A = 1000\,(1 + 0.05)^4$$

$$\Rightarrow A = 1000 \times 1.21550625$$

$$\Rightarrow A = ₹1215.50 \text{ (approx.)}$$

Now, C.I = A − P

$$\Rightarrow C.I = 1215.50 - 1000$$

$$\Rightarrow C.I = ₹215.50$$

Calculation trick:
1.05 × = = =
Then × 1000 which gives 1215.50 as amount
Subtract 1000 from it to get the value of C.I.
Therefore, the amount and C.I are ₹1215.50 and ₹215.50 respectively.

Hence, the correct option is (a).

Example 15. ₹100 will become after 20 years at 5%, p.a compound interest of **(ICAI)**
(a) ₹250 (b) ₹205 (c) ₹165.33 (d) None of these

Sol. (c) It is given that P = ₹100, R = 5% and t = 20
We know that,

$$\text{Compound interest C.I.} = \left(1 + \frac{R}{100}\right)^t - P$$

P = Principal
R = Interest rate
t = Time (in Years)

$$\Rightarrow C.I = 100\left(1 + \frac{5}{100}\right)^{20} - 100$$

$$\Rightarrow C.I = 100 \times 2.65329 - 100$$

$$\Rightarrow C.I = ₹165.33$$

Therefore, the required sum of money is ₹165.33
Hence, the correct option is (c).

PRACTICE QUESTIONS (PART B)

1. The time in which a sum of money will be double at 5% p.a. C.I is
(a) 10 years (b) 12 years (c) 14.2 years (d) None of these

2. If P = ₹8,000, R = 5% p.a., A = ₹8,820, then find the time period if the interest is compounded annually.
(a) 3.1 years (b) 3 years (c) 2.5 years (d) 2 years

3. Mr. X borrowed ₹5,120 at 12.5% p.a. C.I. At the end 3 years, the money was repaid along with the interest accrued. The amount of interest paid by him is.
(a) ₹2100 (b) ₹2170 (c) ₹2000 (d) None of these

4. If ₹64 amount to ₹83.20 in 2 years, what will ₹86 amount to in 4 years at the same rate percent per annum?
(a) 127.60 (b) 147.60 (c) 145.34 (d) 117.60

5. 10 years ago the earning per share (EPS) of ABC Ltd. was ₹5 share. Its EPS for this year is ₹22. Compute at what rate, EPS of the company grows annually? **(Dec 2022)**
(a) 15.97% (b) 16.77% (c) 18.64% (d) 14.79%

Answer Key

1. (c) **2.** (d) **3.** (b) **4.** (c) **5.** (a)

CONVERSION PERIOD

The time period according to which it is going to be compounded.

E.g.: we can take a compounding period–annually, semi-annually, quarterly, monthly or daily.

TYPICAL CONVERSION PERIOD

Conversion Period	Description	Number of Conversion period in a year (c)
1 day	Compounded daily	365
1 month	Compounded monthly	12
3 months	Compounded quarterly	4
6 months	Compounded Semi Annually	2
12 months	Compounded Annually	1

Example 16. ₹5,000 is invested at annual rate of interest of 5%. What is the amount after two years if compounding is done]

(i) Annually (ii) Semi-annually

(iii) Quarterly (iv) monthly

Sol. Given: Amount invested (P) = ₹5,000

Rate of interest (r) = 5%

Time (t) = 2 years

We know that,

$$\text{Amount, } A = P\left(1 + \frac{r}{100c}\right)^{tc}$$

(i) When the compounding is annually i.e., c = 1

$$\text{Thus, Amount (A)} = P\left(1 + \frac{r}{100}\right)^{t}$$

$$A = 5000\left(1 + \frac{5}{100}\right)^{2}$$

$$= 5000\,(1.05)^2 = ₹5,512.5$$

(ii) When the compounding is semi-annually i.e., c = 2

$$\text{Thus, Amount (A)} = P\left(1 + \frac{r}{2 \times 100}\right)^{2t}$$

$$A = 5000\left(1 + \frac{5}{200}\right)^{4}$$

$$= 5000\,(1.025)^4 = ₹5,519.064$$

(iii) When the compounding is done quarterly i.e., c = 4

Thus, Amount (A) = $P\left(1+\frac{r}{4\times 100}\right)^{4t}$

$A = 5000\left(1+\frac{5}{400}\right)^{8} = 5000\,(1.0125)^{8} = ₹5,522.43$

(iv) When the compounding is done monthly i.e., c = 12

Thus, Amount (A) = $P\left(1+\frac{r}{12\times 100}\right)^{12t}$

$A = 5000\left(1+\frac{5}{1200}\right)^{24}$

$= 5000\,(1.00141666) = ₹5,524.706$

FORMULAS FOR COMPOUND INTEREST

- $A_n = P(1 + i)^n$
- Compound Interest (C.I.) = $P[(1 + i)^n - 1]$

where, A_n = Amount accumulated after nth period

i = Annual rate of interest

P = Principal amount

n is total conversion i.e. time period * total no. of conversions

Example 17. The C.I on ₹16000 for $1\frac{1}{2}$ years at 10% p.a. payable half–yearly is **(ICAI)**

(a) ₹2,222 (b) ₹2,522 (c) ₹2,500 (d) None of these

Sol. (b) Given, Principal (P) = ₹16000

Time (t) = $1\frac{1}{2} = \frac{3}{2}$ years

Rate of interest = 10%

n = 2 (interest is payable half yearly)

Compound interest is given as,

$C.I = P\left(1+\frac{R}{n\times 100}\right)^{nt} - P$

$= 16000\left(1+\frac{10}{2\times 100}\right)^{2\times\frac{2}{3}} - 16000$

$= 16000\,(1 + 0.06)^3 - 16000$

$= (16000 \times 1.157625) - 16000$

= ₹18522 – ₹16000 = ₹2522

Hence, the correct option is (b).

Example 18. In what time will ₹4,000 amount to ₹4,410 at 10% per annum interest compounded half-yearly?

(a) 1 year (b) 2 years

(c) 2.5 years (d) Cannot be determined

Sol. (a) Given, P = ₹4000

A = ₹4,410

r = 10%

c = 2

Thus, Amount (A) = $P\left(1+\frac{r}{2\times 100}\right)^{2t}$

$\Rightarrow 4410 = 4000\left(1+\frac{10}{200}\right)^{2t}$

$\Rightarrow \frac{4410}{4000} = (1.05)^{2t}$

$\Rightarrow 1.1025 = (1.05)^{2t}$

$\Rightarrow (1.05)^2 = (1.05)^{2t}$

On comparing, we get

$2t = 2$

$\Rightarrow t = 1$ year

Hence, the correct option is (a).

Example 19. The difference between the S.I and the C.I on ₹2,400 for 2 years at 5% p.a. is

(a) ₹5 (b) ₹10 (c) ₹16 (d) ₹6 **(ICAI)**

Sol. (d) Given, Principal (P) = ₹2,400

Time (t) = 2 years

Rate of interest (R) = 5% p.a

Compound Interest (C.I) = $P\left(1+\frac{R}{100}\right)^{t} - P$

Simple Interest (S.I.) = $\frac{P\times R\times t}{100}$

According to the question,

C.I − S.I

$= \left[P\left(1+\frac{R}{100}\right)^{t} - P\right] - \left(\frac{P\times R\times t}{100}\right)$

$= \left[2400\left(1+\frac{5}{100}\right)^{2} - 2400\right] - \left(\frac{2400\times 5\times 2}{100}\right) = [2400\,(1+0.05)^2 - 2400] - 240$

$= [2400\,(1.1025) - 2400] - 240 = 246 - 240 = ₹6$

Therefore, the required difference is ₹6.

Hence, the correct option is (d).

Example 20. The difference between C.I. and S.I. on a certain sum of money invested for 3 years at 6% p.a is ₹110.16. The Principal is **(ICAI)**

(a) ₹3,000 (b) ₹3,700 (c) ₹12,000 (d) ₹10,000

Sol. (d) Given, Time (T) = 3 years

Rate (R) = 6% p.a

Difference between C.I and S.I. = ₹110.16

n = 3

We know that,

Compound interest is given as,

$$C.I = P\left(1+\frac{R}{100}\right)^n - P$$

Simple interest is given as, $S.I = \frac{P \times R \times T}{100}$

According to the question,

C.I. – S.I = ₹110.16

$$\Rightarrow P\left(\left(1+\frac{6}{100}\right)^3 - 1\right) - \frac{P \times 6 \times 3}{100} = ₹110.16$$

$$\Rightarrow 0.191\,P - 0.18\,P = ₹110.16$$

$$\Rightarrow 0.011\,P = ₹110.16$$

$$\Rightarrow P = \frac{110.16}{0.011}$$

$$\Rightarrow P = ₹10,000 \text{ (approx.)}$$

Hence, the correct option is (d).

PRACTICE QUESTIONS (PART C)

1. In how many years a sum of money triples at 5% pa. compound interest payable on a half- yearly basis?

   (a) 15 years 7 months (b) 18 years 6 months

   (c) 18 years 8 months (d) 22 years 3 months

2. Find the compound interest of ₹700 invested for 15 years at 8% compounded semiannually.

   (a) ₹1570.37 (b) ₹2270.37 (c) ₹1800.75 (d) None of these

3. How long will it take for ₹5000 amount to ₹7000 if it is invested at 8% compounded quarterly?

   (a) $4\frac{1}{4}$ years (b) $3\frac{1}{2}$ years (c) $5\frac{1}{4}$ years (d) None of these

4. At what rate per annum compounded annually will ₹20,000 amount to ₹23,254 in 2 years?

(a) 10% (b) 8% (c) 6.07% (d) 5.6%

5. The difference between the simple and compound interest on a certain sum for 2 years at 4% p.a. is ₹600. The sum is

(a) ₹3,50,000 (b) ₹3,75,000 (c) ₹3,27,500 (d) ₹2,97,550

6 A person deposits ₹4000 in a bank which pays an interest of 5% per annum compounded continuously. How much will be in his account after 4 years?

(a) ₹3400.50 (b) ₹4885.60 (c) ₹1500.45 (d) None of these

7. A person opened an account on Jan 2019 with a deposit of ₹500. The account paid 5% interest compounded quarterly. In July 2019 he closed the account and added enough additional money to invest in a 6-months time deposit for ₹800, earning 5% compounded monthly.

(I) How much additional amount did the person invest on July 1?

(II) What was the maturity value of his time deposit on Jan 1, 2020?

(III) How much total interest was earned?

(a) ₹512.58, ₹820.21, ₹32.79 (b) ₹512.58, ₹820.21, ₹12.58

(c) ₹237.42, ₹820.21, ₹32.79 (d) ₹1037.42, ₹820.21, ₹32.79

8. The annual birth and death rates per 1,000 are 30 and 10 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is

(a) 35 years (b) 30 years (c) 25 years (d) none of these

Answer Key							
1. (d)	2. (a)	3. (a)	4. (c)	5. (b)	6. (b)	7. (c)	8. (a)

EFFECTIVE RATE OF INTEREST

- If interest is compounded more than once a year, the effective interest rate for a year exceeds the per annum interest rate.
- For example: If we invest ₹1000 for a year at the compound interest rate of 5% quarterly. The effective rate of interest will be more when compounded in comparison to 5%.

  So, to find out effective interest rate:

  Effective Rate of Interest (E) is given as

$$E = \left(\left(1 + \frac{r}{100c} \right)^{c} - 1 \right) \times 100\%$$

Example 21. The effective rate of interest corresponding to a nominal rate 4% p.a. payable half yearly is

(a) 4% (b) 4.04% (c) 4.45% (d) None of these

Sol. (b) Given, Rate of interest (R) = 4% p.a.

Interest is payable half yearly, i.e., c = 2

r = 4%

Effective Rate of Interest (E) is given as $\left(\left(1+\frac{r}{100c}\right)^{c}-1\right)\times 100\%$

$\Rightarrow E = \left(\left(1+\frac{4}{200}\right)^{2}-1\right)\times 100\%$

$= ((1.02)^2 - 1) \times 100\% = (1.0404 - 1) \times 100\% = 0.0404 \times 100\% = 4.04\%$

Hence, the correct option is (b).

Example 22. Find the compound interest and effective rate of interest if an amount of ₹10,000 is deposited in a bank for one year at the rate of 4% per annum compounded quarterly.

(a) ₹406, 4% (b) ₹406, 4.06% (c) ₹400, 4.50% (d) None of these

Sol. (b) Given, P = ₹10,000

r = 4%, c = 4

t = 1 year

Thus, C.I. $= \left[P\left(1+\frac{r}{4\times 100}\right)^{4t}-1\right]$

$= 10{,}000\left[\left(1+\frac{4}{400}\right)^{4\times 1}-1\right] = 10{,}000\,[(1.01)^4 - 1]$

$= 10000(0.04060401) = 406$ (approx.)

Now, effective rate of interest can be calculated as:

E.R.I. $= \left[\left(1+\frac{4}{4\times 100}\right)^{4\times 1}\right]\times 100 = [(1.01)^4 - 1] \times 100$

$= [1.04060401 - 1] \times 100 = 4.06$ % (approx.)

Hence, the correct option is (b).

Example 23. Which is a better investment: 7% per year compounded monthly or 7.5% per year compound interest?

(a) 7% compound interest (b) 7.5% compound interest

(c) Both are same (d) Cannot be determined

Sol. (b) Given that,

Number of conversion period, c = 12

Nominal rate of interest, r = 7%

Therefore, effective rate of interest $E = \left[\left(1+\frac{r}{100c}\right)^{c}-1\right]\times 100$

$$\Rightarrow E = \left[\left(1 + \frac{7}{1200}\right)^{12} - 1\right] \times 100$$

$\Rightarrow E = [1.072290 - 1] \times 100$

$\Rightarrow E = 0.072290 \times 100$

$\Rightarrow E = 7.229\%$ (approx.)

As we know, $7.5\% > 7.229\%$

Thus, 7.5% per year compound interest is a better investment than 7% per year compounded monthly.

Hence, option (b) is the correct answer.

Example 24. A machine is depreciated at the rate of 10% on reducing balance. The original cost of the machine was ₹50,000 and its ultimate scrap value was ₹15,000. The effective life of the machine is

(a) 10 years (b) 11.4 years (c) 12.5 years (d) None

Sol. (b) Given, Depreciation Rate = 10%

Principal (Initial Value) = ₹50,000

Amount (Net Value) = ₹15,000

$$\therefore i = \frac{\text{Rate of Depreciation}}{100} = \frac{10}{100} = 0.10$$

We know that, Net value = Initial value $\times (1 - i)^n$

$\Rightarrow 15000 = 50000(1 - 0.1)^n$

$\Rightarrow (0.3) = (0.9)^n$

Taking log on both sides, we get

$\log 0.3 = \log (0.9)^n$

$\Rightarrow \log 0.3 = n \log 0.9$ [$\because \log am = m \log a$]

$$\Rightarrow n = \frac{\log 0.3}{\log 0.9}$$

$\Rightarrow n = 11.4$ years (appx.)

Hence, the correct answer is option (b).

PRACTICE QUESTIONS (PART D)

1. The effective rate of interest corresponding to a nominal rate 3% p.a. payable half yearly is
(a) 3.2% p.a. (b) 3.25% p.a. (c) 3.0225% p.a. (d) None of these

2. The effective annual rate of interest corresponding to a nominal rate of 6.5% p.a. compounded quarterly is
(a) 6.0% (b) 6.25% (c) 6.50% (d) 6.66%

3. An investment earns 5% interest per annum, compounded half yearly. What is the effective annual rate of interest?
(a) 5.0650% (b) 5.0925% (c) 5.0625% (d) 5.1525%

4. Which is the better investment, 9% compounded quarterly or 9.2% per year simple interest?

   (a) 9% compounded (b) 9.2% compound interest

   (c) Both are same (d) Cannot be determined

5. How long will ₹8,000 take to amount to ₹9,261 at 20% per annum, if the interest is compounded quarterly?

   (a) 6 months (b) 8 months (c) 9 months (d) 1 year

6. A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹1,00,000 and its ultimate scrap value was ₹30,000. The effective life of the machine is

   (a) 4.5 years (appx.)(b) 5.4 years (appx.)(c) 5 years (appx.) (d) None of these

7. A machine worth ₹4,90,740 is depreciated at 15% on its opening value each year. When it value would reduce to ₹2,00,750?

   (a) 5 year 5 months(b) 5 year 6 months(c) 5 year 7 months (d) 5 year 8 months

Answer Key

1. (c) **2.** (d) **3.** (c) **4.** (a) **5.** (c) **6.** (b) **7.** (a)

ANNUITY

In many cases, individuals have regular financial obligations where they make equal payments at fixed intervals, such as monthly or yearly. Examples of such payments include life insurance premiums, rent for a rented house, housing loan payments, vehicle loan instalments, etc. This consistent payment of a fixed amount over a specified time period is known as an annuity.

An annuity can be defined as a sequence of periodic payments or receipts made regularly over a specific duration. It applies to situations where individuals receive a fixed amount regularly, like pension payments or rental income from a property.

TAKING THREE CASES

Case 1: When the payment or receipt amount is not the same.

Year end	Payment (₹)
2014	1,000
2015	2,000
2016	3,000
2017	4,000

Case 2: When Payment (or received) amount is same, but the period is not constant.

Year end	Payment (₹)
2014	1,000
2015	1,000

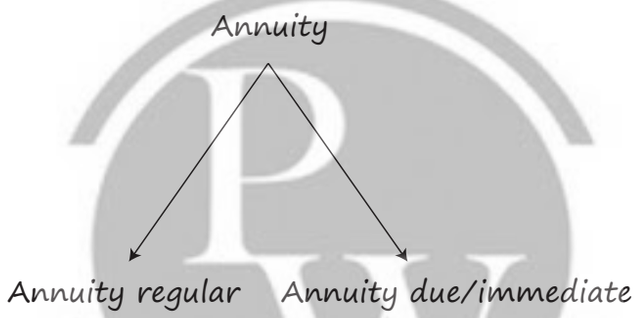
Year end	Payment (₹)
2018	1,000
2020	1,000

Case 3: When payment and periodic period are same.

Year end	Payment (₹)
2014	2,000
2015	2,000
2016	2,000
2017	2,000

Here, case 1 and 2 cannot be an annuity whereas case 3 can be an annuity since the payment and the time interval between the consecutive payments are same.

There are two types of Annuity:



REGULAR ANNUITY

In a regular annuity, the first payment or receipt occurs at the end of the first period. For example, consider the following table:

Year end	Payment (₹)
2014	2,000
2015	2,000
2016	2,000
2017	2,000

In this case, the first payment or receipt takes place at the end of the first year, making it a regular annuity.

ANNUITY DUE OR ANNUITY IMMEDIATE

In an annuity due or annuity immediate, the first payment or receipt is made today, at the beginning of the annuity. For example, consider the following table:

In the beginning of	Payment (₹)
2014	2,000

In the beginning of	Payment (₹)
2015	2,000
2016	2,000
2017	2,000

In this case, the first payment or receipt is made at the beginning of the first year. This type of annuity is referred to as annuity due or annuity immediate.

FUTURE VALUE

Future value refers to the monetary worth of an investment or sum of money at a specific point in the future. It represents the value that today's money will grow to over time when compounded at a certain interest rate.

We know that,

$$An = P(1 + i)^n$$

Future value of a single cash flow can be computed by the above formula. Replace A by future value (F) and P by single cash flow (C.F.), therefore

$$F = C.F. (1 + i)^n$$

Let's consider an example to illustrate the concept of future value. Suppose you invest ₹10,000 in a fixed deposit that offers an annual interest rate of 5%. At the end of the first year, your investment will grow to ₹10,500. This amount includes the original principal of ₹10,000 and the interest earned of ₹500. Therefore, we can say that ₹10,000 invested today is expected to be worth ₹10,500 in one year's time, assuming an interest rate of 5%.

FUTURE VALUE OF AN ANNUITY REGULAR

Example 25. You invest ₹1000 in a three year investment that pays you 6 % per annum. Calculate the future value of the investment.

(a) ₹1191.1 (b) ₹1200.5 (c) ₹900.8 (d) ₹1000

Sol. (a) Given, Single cash flow (C.F.) = ₹1000

Rate of interest (i) = 6% p.a. = 0.06 p.a.

Time period = 3 years

We know that,

$F = C.F. (1 + i)^n$

Where, $i = \frac{r}{100 \times c}$ and n = t.c.

Thus, $F = C.F. (1 + i)^n$

where, $i = \frac{r}{100 \times c}$ and n = t.c

Thus, $F = C.F. (1 + i)^n$

$= 1000 (1 + 0.06)^3 = 1000 (1.06)^3 = 1191.1$ (approx.)

Calculator trick:

1. 1.06 × = =

2. Then × 1000

Therefore, the future value of the investment is ₹1191.1.

Hence, the correct option is (a).

CALCULATING THE FORMULA FOR FUTURE VALUE OF ANNUITY

If A be the periodic payments, the future value A(n, i) of the annuity is given by

$$A(n, i) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Example 26. Find the future value of an annuity of ₹10000 made annually for 5 years at an interest rate of 12% compounded annually.

Given that $(1.12)^5 = 1.7623$.

(a) ₹52500.64 (b) ₹60000.50 (c) ₹63528.47 (d) None of these

Sol. (c) Given, Annuity amount (A) = ₹10,000

Time in years (n) = 5×1 = 5

Rate (i) = 12% = $\frac{12}{100}$ = 0.12 p.a.

Future value FV is given as,

$$FV = \frac{A\{(1+i)^n - 1\}}{i} = \frac{10000\ \{(1+0.12)^5 - 1\}}{0.12}$$

$$= \frac{10000\ \{(1.12)^5 - 1\}}{0.12} = \frac{10000\ (1.7623 - 1)}{0.12}$$

$$= \frac{10000\ (0.7623)}{0.12} = ₹63528.47$$

Calculator trick:

1. 1.12 × = = = =

2. −1

3. × 10000

4. ÷0.12

Hence, the correct answer is option (c).

Example 27. ₹500 is invested at the end of each month in an account paying interest 12% per year compounded monthly. What is the future value of this annuity after the 9th payment?

(a) ₹4000 (b) ₹4684.36 (c) ₹5526.64 (d) None of these

Sol. (b) Given, Annuity amount (A) = ₹500

c = 12

n = 1 × 9 = 9

Rate (i) = 12% = $\frac{12}{100 \times 12}$ = 0.01 (monthly interest rate)

Future Value is given as,

$$A(9,0.01) = \frac{A\left\{(1+i)^n - 1\right\}}{i} = \frac{500\left\{(1+0.01)^9 - 1\right\}}{0.01}$$

$$= \frac{500\left\{(1.01)^9 - 1\right\}}{0.01} = ₹4684.36$$

Calculator trick:

1. 1.01 × = = = = = = = = =
2. −1
3. ÷0.01
4. ×500

Hence, the correct option is (b).

FUTURE VALUE OF ANNUITY DUE OR ANNUITY IMMEDIATE

To calculate the future value of an Annuity due/Annuity immediate:

Step 1: Calculate the future value as though it is an ordinary annuity.

Step 2: Multiply the result by (1 + i)

Example 28. Rahul invests ₹10,000 every year starting from today for next 10 years. Suppose interest rate is 8% per annum compounded annually. Calculate future value of the annuity.

Given that $(1 + 0.08)^{10} = 2.15892500$

(a) ₹1,56,455 (b) ₹1,60,855 (c) ₹1,90,865 (d) ₹2,00,505

Sol. (a) Given, P = ₹10,000, n = 10 × 1 = 10 and i =8% = 0.08

Amount of Annuity is given as,

$$A = \frac{P((1+i)^n - 1)}{i}$$

$$= \frac{10,000((1+0.08)^{10} - 1)}{0.08} = 1,44,865.625$$

Now, multiply the above result by (1 + i) to get the required result i.e.,

= 1,44,865.625(1 + 0.08) = 1,44,865.625(1.08) = 1,56,454.875

Therefore, the future value of the annuity is ₹1,56,454.875.

Hence, the correct option is (a).

Example 29. The amount of an annuity certain of ₹150 for 12 years at 3.5% p.a C.I is

(a) ₹2,190.28 (b) ₹1,290.28 (c) ₹2,180.28 (d) None of these

Sol. (a) Given, Initial Amount (P) = ₹150

Time (n) = 12 × 1 = 12

Interest (i) = 3.5% = 0.035

Amount of Annuity is given as,

$$A = \frac{P((1+i)^n - 1)}{i} = \frac{150\ ((1+0.035)^{12} - 1)}{0.035}$$

$$= \frac{150(1.511 - 1)}{0.035} = \frac{76.65}{0.035} = ₹2190.28 \text{ (approx.)}$$

Therefore, the required amount is ₹2190.28.

Hence, the correct option is (a).

Example 30. If the amount of an annuity after 25 years at 5% p.a. C.I is ₹50,000, the annuity will be

(a) ₹1,406.90 (b) ₹1,047.62 (c) ₹1,146.90 (d) None of these

Sol. (b) Given, Future Value (FV) = ₹50000

Time in years (n) = 25 × 1 = 25

$$\text{Interest rate (i)} = 5\% = \frac{5}{100 \times 1} = 0.05$$

Future Value FV is given as,

$$FV = \frac{a\{(1+i)^n - 1\}}{i}$$

$$50000 = \frac{a\{(1+0.05)^{25} - 1\}}{0.05}$$

$$50000 = \frac{a\{2.386\}}{0.05}$$

$$a = \frac{50000 \times 0.05}{2.386}$$

a = ₹1047.62

Therefore, the required annuity will be ₹1047.62.

Hence, the correct option is (b).

Example 31. A person invests ₹500 at the end of each year with a bank which pays interest at 10% p.a. C.I. annually. The amount standing to his credit one year after he has made his yearly investment for the 12th time is

(a) ₹11,761.36 (b) ₹10,000 (c) ₹12,000 (d) none of these

Sol. (a) Given, Annuity amount (a) = ₹500

Time (n) = 12

$$\text{Interest rate (i)} = 10\% = \frac{10}{100 \times 1} = 0.1$$

Future Value is given as,

$$A(12, 0.1) = \frac{a\{(1+i)^n - 1\}}{i} \times (1+i) = \frac{500\{(1+0.1)^{12} - 1\}}{0.1} \times (1+0.1)$$

$= \frac{500\{2.1384283767\}}{0.1} \times (1.1) = (10690.1418835)\,(1.1) = ₹11761.36$ (approx.)

Therefore, the required amount is ₹11761.36.

Hence, the correct answer is option (a).

PRACTICE QUESTIONS (PART E)

1. Find the future value of annuity of ₹6,000 made annually for 8 years at interest rate of 12% compounded annually.

   (Given that $(1.12)^8 = 2.47596$

   (a) ₹73798 (b) ₹53655 (c) ₹87561 (d) ₹98934

2. At an interest rate of 14% compounded annually, how much should be invested at the end of each year for a period of 7 years to accumulate a total amount of ₹11,000 by the end of the 7th year?

   (a) ₹950 (b) ₹1400 (c) ₹1030 (d) ₹2000

3. Sahil decided to invest ₹50,000 every year starting from today for next 12 years. If the interest rate is 7.5% per annum compounded annually, then calculate the future value of the annuity.

   [Given that $(1 + 0.075)^{12} = 2.3817796$]

   (a) ₹990,275 (b) ₹900,298 (c) ₹890,275 (d) None of these

4. Raju invests ₹20,000 every year in a deposit scheme starting from today for next 12 years. Assuming that interest rate on this deposit is 7% per annum compounded annually. What will be the future value of this annuity? Given that $(1 + 0.07)^{12} = 2.25219150$

   (a) ₹540,576 (b) ₹382,813 (c) ₹643,483 (d) ₹357,769

Answer Key

1. (a) 2. (c) 3. (a) 4. (b)

PRESENT VALUE

WE UNDERSTOOD WHAT THE FUTURE VALUE IS!

- The value of today's money in the future, right!!
- Now, present value means: Value of future money in the present.

  Present Value refers to the current value of a future sum of money, considering the time value of money and the expected rate of return. It represents the amount of money that would need to be invested today to equal the future amount at a given interest rate.

  So, formula for Present value for amount A_n, time period n period at the rate of interest per period i can easily be obtained from the below equation:

$$An = P(1 + i)^n$$

$$\text{i.e., } P = \frac{A_n}{(1 + i)^n}$$

PRESENT VALUE OF ANNUITY REGULAR

The present value of an annuity regular refers to the current worth of a series of periodic payments or receipts over a specified time period. It helps determine the value of the annuity in today's terms.

To calculate the present value of an annuity regular, we can use the following formula:

$$P.V. = A \times \frac{(1+i)^n - 1}{i \times (1+i)^n} = A.P(n,i)$$

Where, $P(n,i) = \frac{(1+i)^n - 1}{i \times (1+i)^n}$

One more formula is $A = \frac{V}{P(n,i)}$ and it uses amortization, where amortization refers to the process of gradually paying off a debt over a specific period of time through regular instalments. It involves the systematic reduction of the principal amount owed, along with the payment of interest, until the debt is fully repaid.

In case of loan if you pay your annuity properly i.e. principal amount + interest, instalment are same and paid over equal period of time then it is said that loan is amortized.

For example, let's consider an annuity regular where you receive ₹5,000 at the end of each year for five years, and the interest rate is 6% per annum. Using the formula, we can calculate the present value of this annuity regular.

Example 32. The present value of an annuity of ₹80 a year for 20 years at 5% p.a is

(a) ₹997 (b) ₹900 (c) ₹1,000 (d) none of these

Sol. (a) $P.V. = A \times \frac{(1+i)^n - 1}{i \times (1+i)^n}$, where

A = value of each payment

i = rate of interest per period

n = number of periods

Given,

A = ₹80,

i = 5% p.a. = 0.05 p.a.

n = 20 × 1 = 20

Thus,

$$P.V. = 80 \times \frac{(1+0.05)^{20} - 1}{0.05 \times (1+0.05)^{20}}$$

$$P.V. = 80 \times \frac{(1.05)^{20} - 1}{0.05 \times (1.05)^{20}}$$

P.V. = 996.97

P.V. = 997 (approx.)

Calculator trick:

1. Start from MRC
2. Find $(1.05)^{20}$ i.e., 1.05 × = = = = = ... (21 steps)
3. Click M+ to store the value
4. Now, subtract 1
5. ÷0.05
6. ÷MRC
7. ×80

In short find $(1.05)^{20}$ – 1 ÷ 0.05 ÷ MRC = then × 80

Therefore, the present value of an annuity is ₹997.

Hence, the correct answer is option (a) i.e. ₹997.

Example 33. A person bought a house paying ₹20,000 cash down and ₹4,000 at the end of each year for 25 years at 5% p.a. C.I. The cash down price is **(ICAI)**

(a) ₹75,000 (b) ₹76,000 (c) ₹76,370.80 (d) None of these

Sol. (c) Given, ₹20,000 is paid at the beginning as cash down and the rest ₹4,000 is made in the installments.

Thus, the present value is calculated by:

$$P.V. = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right], \text{ where}$$

$A = ₹4000$, $n = 25 \times 1 = 25$ and $i = 5\% = \frac{5}{100 \times 1} = 0.05$

$$\Rightarrow P.V. = 4000\left[\frac{(1+0.05)^{25} - 1}{0.05(1+0.05)^{25}}\right]$$

$$\Rightarrow P.V. = 4000\left[\frac{(0.05)^{25} - 1}{0.05(1+0.05)^{25}}\right]$$

$$\Rightarrow P.V. = 4000\left(\frac{3.386354 - 1}{0.05(3.386354)}\right)$$

$$\Rightarrow P.V. = 56375.5$$

Calculator trick:

Find $(0.05)^{25}$ and store it then −1 ÷ 0.05 ÷ MRC = ×4000

Therefore, the cash down price = 20000 + 56375.5 = 76375.5

Hence, the correct answer is option (c) i.e., ₹76375.5.

PRESENT VALUE OF ANNUITY DUE OR ANNUITY IMMEDIATE

Steps to calculate Present Value for Annuity Immediate/Due

1. Calculate the present value for (n – 1) period as annuity regular
2. Add the initial payment (receipt) in the above.

Example 34. Your Papa wants to give you ₹10000 every year starting from today for next 5 years as a gift. So, being a smart father, he invested at the interest rate of 15% in mutual funds today morning. What should be the exact amount to be invested such that it exactly gives the money to be gifted?

(a) ₹50,000.0 (b) ₹28,549.8 (c) ₹38,549.8 (d) None of these

Sol. (c) Since, it is an annuity immediate. Thus,

A = ₹10000

n = 4 years

i = 15% = 0.15

Therefore, P.V. $= A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$= 10000\left[\frac{(1+0.15)^4 - 1}{0.15(1+0.15)^4}\right] = 10000\left[\frac{(1.15)^4 - 1}{0.15(1.15)^4}\right]$ = ₹28549.8

Now, the total amount to be invested = 28,549.8 + 10,000 = ₹38,549.8

Hence, the correct option is (c).

Example 35. Suppose you bought a flat worth ₹40,00,000 at the interest rate of 5% where you need to pay an equal 10 instalments at the end of every year where initially you gave to the bank ₹10,00,000. So, what will be the amount of every instalment?

(a) ₹97,128.25 (b) ₹2,85,900.800

(c) ₹9,71,000.0 (d) ₹3,88,513.725

Sol. (d) According to the given information,

Cost of the flat = ₹40,00,000

Interest rate = 5%

Number of instalments = 10

Since, the initial payment of ₹10,00,000 is already made, thus the remaining amount is ₹30,00,000.

P.V. = ₹30,00,000

n = 10

i = 5% = 0.05

Therefore, P.V. $= A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

$30,00,000 = A\left[\frac{(1+0.05)^{10} - 1}{0.05(1+0.05)^{10}}\right]$

$30,00,000 = A\left[\frac{(1.05)^{10} - 1}{0.05(1.05)^{10}}\right]$

A = ₹3,88,513.725

Therefore, the amount of every instalment is ₹3,88,513.725.

Example 36. If Aisha borrows ₹6,00,000 from City Bank at the interest rate of 12% for 5 years. What will be the equal annual instalment?

(a) ₹2,66,000.750 (b) ₹1,66,445.839

(c) ₹5,54,655.395 (d) None of these

Sol. (b) According to the given information,

Amount borrowed = ₹6,00,000

Interest rate = 12%

Number of instalments = 5

P.V. = ₹6,00,000

$n = 5$

$i = 12\% = 0.12$

We know that,

$$P.V. = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$6,00,000 = A\left[\frac{(1+0.12)^5 - 1}{0.12(1+0.12)^5}\right]$$

$$6,00,000 = A\left[\frac{(1.12)^5 - 1}{0.12(1.12)^5}\right]$$

A = ₹1,66,445.839

Therefore, the amount of every instalment is ₹1,66,445.839.

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART F)

1. What is the present value of ₹1 to be received after two years compounded annually at 10% interest rate?

   (a) ₹1 (b) ₹0.83 (c) ₹1.5 (d) ₹2

2. Find the present value of ₹10,000 to be required after 5 years if the interest rate be 9%?

   (a) ₹6,500 (b) ₹6,000 (c) ₹8,500 (d) None of these

3. Find the present value of ₹5,00,000 be required after 6 years if the rate of interest is 7.5%. (Given that $(1.075)^6 = 1.5433$)

   (a) ₹303,981 (b) ₹323,981 (c) ₹300,781 (d) ₹360,900

4. Find the present value of ₹1000 due at the end of 4 years if money is 7% effective.

   (a) ₹616.56 (b) ₹812.75 (c) ₹762.89 (d) None of these

5. Calculate the present value of an annuity of ₹2,000 payable at the end of each year for a period of 5 years. The rate of interest is 8% compounded annually.

   (a) ₹7,985 (approx) (b) ₹8,360 (approx)

   (c) ₹12,000 (approx) (d) None of these

6. A loan of ₹1,02,000 is to be paid back in two equal annual instalments. If the rate of interest is 4% p.a. compounded annually, then the total interest charged under this instalment plan is

(a) ₹6,160 (b) ₹8,120 (c) ₹5,980 (d) ₹7,560

7. Sachin purchases a bike for ₹3,50,000. He decided to get a loan of ₹3,00,000 at 15% p.a. from a bank and remaining balance of ₹50,000 he pays at the time of purchase. He has to pay the whole amount of loan in 12 equal monthly installments with interest starting from the end of the first month. The money he has to pay at the end of every month is **[Given: $(1.0125)^{12} = 1.16075452$]**

(a) ₹27077.50 (b) ₹37798.50 (c) ₹54000.50 (d) None of these

Answer Key

1. (b) **2.** (a) **3.** (b) **4.** (c) **5.** (a) **6.** (a) **7.** (a)

FEW MORE USE OF TIME VALUE OF MONEY

- Sinking funds
- Leasing
- Capital expenditure
- Valuation of bond
- Perpetuity
- Net present value technique (NPV)

SINKING FUNDS

Sinking funds refer to setting aside money over a period of time to accumulate a specific amount needed to repay a debt or fund a future obligation like debentures, renew items etc.

It involves making regular payments into an account or investment to ensure sufficient funds are available when needed.

Example: A homeowner setting aside a certain amount of money each month in a savings account to accumulate funds for a future major home repair, such as replacing the roof or renovating the kitchen.

Thus, using formula A = P.A. (n, i); where $A(n,i) = \left[\frac{(1+i)^n - 1}{i}\right]$

Example 37. Company Alpha needs to pay ₹10,00,000 to banks after 5 years for which they started investing every year at the end with equal instalment with an interest rate of 10%.

(a) ₹1,00,000 (b) ₹2,63,785.64 (c) ₹1,63,797.48 (d) None of these

Sol. (c) According to the given information,

Future value = ₹10,00,000

$n = 5 \times 1 = 5$

$i = 10\% = 0.10$

We know that,

$$F.V. = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow 10,00,000 = A\left[\frac{(1+0.10)^5 - 1}{0.10}\right]$$

$$\Rightarrow 10,00,000 = A\left[\frac{(1.10)^5 - 1}{0.10}\right]$$

$\Rightarrow$ A = ₹1,63,797.48

Therefore, the required equal instalment is ₹1,63,797.48

Hence, the correct option is (c).

Example 38. A company creates a sinking fund of ₹2,00,000 in a bank account for 15 years for which the bank offers interest rate 6% per annum. The yearly payment to be paid by company is approximately

(a) ₹8,945 (b) ₹8,145 (c) ₹8,593 (d) ₹9,645

Sol. (c) According to the given information,

Future value = ₹2,00,000

$n = 15 \times 1 = 15$

$i = 6\% = 0.06$

We know that,

$$F.V. = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow 2,00,000 = A\left[\frac{(1+0.06)^{15} - 1}{0.06}\right]$$

$$\Rightarrow 2,00,000 = A\left[\frac{(1.06)^{15} - 1}{0.06}\right]$$

$\Rightarrow$ A = ₹8,592.55

$\Rightarrow$ A = ₹8,593 (approx)

Therefore, the required yearly payment to be paid by company is ₹8,593.

Hence, the correct option is (c).

Example 39. A machine costs ₹5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of ₹25000. What amount should be set aside every year if the sinking fund investment accumulate at 3.5% compound interest p.a.?

(a) ₹16,000 (b) ₹16,500 (c) ₹16,050 (d) ₹16,005

Sol. (c) Given, Cost of machine = ₹5,20,000

Time (n) = 25 years

c = 1

$r = 3.5\%$

$\Rightarrow i = \frac{3.5}{100} = 0.035$

Since, cost of new model is 25% higher, after 25 years with scrap value realization of ₹25000 thus

Total cost = 5,20,000 + 25% of 5,20,000 − 25000 = ₹6,25,000

If 'A' be the amount which is kept aside then

$$625000 = A\frac{\left[(1+0.035)^{25}-1\right]}{0.035}$$

$\Rightarrow$ A = ₹16046.27 = 16,050 (approx.)

Hence, the correct option is (c).

PRACTICAL QUESTIONS (PART G)

1. A sinking fund is created for redeeming debentures worth ₹5 lakhs at the end of 25 year. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a.?

   (a) ₹12,006 (b) ₹12,040 (c) ₹12,039 (d) ₹12,035

2. Sinking fund factor is the reciprocal of:

   (a) Present value interest factor of a single cash flow

   (b) Present value interest factor of an annuity

   (c) Future value interest factor of an annuity

   (d) Future value interest factor of a single cash flow

Answer Key

1. (a) 2. (c)

LEASING

Leasing is a financial arrangement in which the owner of an asset, known as the lessor, grants the right to use the asset to another party, known as the lessee, for a specified period of time. The lessee pays a periodic fee, known as the lease rental, to the lessor for the use of the asset.

In a lease agreement, the lessor retains ownership of the asset while providing the lessee with the right to use it. The asset can be tangible, such as equipment, machinery, vehicles, or real estate, or intangible, such as intellectual property or software.

Example 40. ABC Ltd. wants to lease out an asset costing ₹3,60,000 for a five year period. It has fixed a rental of ₹1,05,000 per annum payable annually starting from the end of first year. Suppose rate of interest is 14% per annum compounded annually on which money can be invested by the company. Is this agreement favourable to the company?

(a) The agreement is favourable (b) The agreement is not favourable

(c) No difference (d) Cannot be determined

Sol. (a) To find the present value of annuity of ₹1,05,000 for five years at the interest rate of 14% per annum.

$$\text{i.e., } A = ₹1,05,000,\ i = 14\% = \frac{14}{100} = 0.14, n = 5 \times 1 = 5$$

$$\text{Thus, P.V.} = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 1,05,000 \times \left[\frac{(1+0.14)^5 - 1}{0.14(1+0.14)^5}\right]$$

$$= 1,05,000 \times \left[\frac{(1.14)^5 - 1}{0.14(1.14)^5}\right] = ₹3,60,473.5$$

Clearly, it is greater than the cost of the asset thus leasing is favorable to the lessor.

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART H)

1. A company is considering of buying two electronics either by making full payment of ₹17,000 or by leasing it for five years at an annual rate of ₹5000. Which course of action is preferable if the company can borrow money at 14% compounded annually?
   (a) Buying the electronics by making a full payment of ₹17,000 is preferable.
   (b) Leasing the electronics for five years at an annual rate of ₹5000 is preferable.
   (c) Both options are equally preferable.
   (d) It cannot be determined based on the given information.
2. A company is considering buying a piece of equipment for ₹75,000 or leasing it for 5 years at an annual rent of ₹18,000. If the company's cost of capital is 12%, which option is more financially attractive?''
   (a) Leasing the equipment is more financially attractive
   (b) Buying the equipment is more financially attractive
   (c) Both options have the same cost
   (d) Cannot be determine

Answer Key

1. (a) 2. (a)

CAPITAL EXPENDITURE

Capital expenditure refers to the funds spent by a company or organization to acquire or improve long-term assets that are expected to generate benefits over an extended period. It involves investing in assets that have a useful life beyond the current accounting period and are not intended for immediate resale.

The purpose of capital expenditure is to enhance the productive capacity, efficiency, or competitiveness of a business. These expenditures are aimed at acquiring or upgrading assets such as property, plant, equipment, machinery, technology, or infrastructure that will support the company's operations and contribute to future revenue generation.

E.g.: A manufacturing company may invest in new machinery to increase production capacity and improve efficiency. A technology company may invest in research and development to develop innovative products or acquire intellectual property rights. A real estate developer may invest in acquiring land and constructing new properties.

Example 41. A cloth making machine can be purchased for ₹200000. Machine will contribute ₹50000 per year for the next five years. Assume borrowing cost is 10% per annum compounded annually. Determine whether the machine should be purchased or not.

(a) Leasing is preferable (b) Should be purchased

(c) No difference (d) Cannot be determined

Sol. (a) Given, Annuity amount (A) = ₹50,000

Rate of interest (i) = 10 % = $\frac{10}{100} = 0.10$

Time (n) = 5 × 1 = 5 years

We know that,

If the present value is less than the purchase value, then the leasing is preferable.

Present value (P.V.) is given as,

$$P.V. = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 50,000\left[\frac{(1+0.10)^5 - 1}{0.10(1+0.10)^5}\right] = 50,000\left[\frac{(1.10)^5 - 1}{0.10(1.10)^5}\right] = ₹1,89,539.338$$

We have concluded that leasing is preferable.

Hence, the correct answer is option (a) i.e., Leasing is preferable.

Example 42. Juicer A with a useful life of seven years costs ₹15,000 while another Juicer B with a useful life of five years costs ₹12,000. The first juicer saves labour expenses of ₹3,000 annually and the second one saves labour expenses of ₹3,200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

(a) Buying Juicer A is the preferred course of action.

(b) Buying Juicer B is the preferred course of action.

(c) Both options are equally preferred.

(d) It cannot be determined based on the given information.

Sol. (b) Cost of juicer A = ₹15,000

Cost of juicer B = ₹12,000

Now, finding the present value of savings of both the juicers.

For juicer A:

A = ₹3,000

$i = 10\% = \frac{10}{100} = 0.10$

n = 7 × 1 = 7

Thus, P.V. $= A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 3000\left[\frac{(1+0.10)^5 - 1}{0.10(1+0.10)^5}\right]$

$= 3000\left[\frac{(1.10)^5 - 1}{0.10(1.10)^5}\right] = ₹14,605$ (approx).

For juicer B:

A = ₹3,200

$i = 10\% = \frac{10}{100} = 0.10$

$n = 5 \times 1 = 5$

Thus, P.V. $= A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 3200\left[\frac{(1+0.10)^5 - 1}{0.10(1+0.10)^5}\right]$

$= 32000\left[\frac{(1.10)^5 - 1}{0.10(1.10)^5}\right] = ₹12,130$ (approx.)

Since, juicer A costing ₹15,000 saves ₹14,605 whereas juicer B costing ₹12,000 saves ₹12,130 thus juicer B is the preferred course of action.

Hence, the correct option is (b).

VALUATION OF BOND

Valuation of a bond refers to the process of determining the fair value or price at which a bond should be bought or sold in the market. Bonds are debt securities issued by governments, municipalities, or corporations to raise capital. When an investor buys a bond, they are essentially lending money to the issuer in exchange for periodic interest payments and the return of the principal amount at maturity.

When we say that bonds are "issued," it means that the bond issuer creates and offers the bonds to the market for purchase by investors. The issuer, which can be a government entity, a local municipality, or a corporate entity, issues the bonds as a means of borrowing money from investors. By selling bonds, the issuer receives an upfront sum of money from investors, which it agrees to repay at a future date, along with periodic interest payments.

Example 43. Rajesh intends purchasing a three year ₹1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and Rajesh requires a rate of return of 20%?

(a) ₹900.35 (b) ₹789.35 (c) ₹1000 (d) None of these

Sol. (b) Here, 10% of ₹1,000 $= \frac{10}{100} \times 1000 = ₹100$

Thus, Present value of bond $= \frac{100}{(1+0.20)} + \frac{100}{(1+0.20)^2} + \frac{100}{(1+0.20)^3} + \frac{1000}{(1+0.20)^3}$

$= \frac{100}{(1.2)} + \frac{100}{(1.2)^2} + \frac{100}{(1.2)^3} + \frac{1000}{(1.2)^3} = 789.35$

Therefore, the purchase value of bond ₹789.35.

Alternative Solution:

Valuation of bond = Present value of A for n periods + Present value of B for n periods

$$= \frac{A((1+i)^n - 1)}{i(1+i)^n} + \frac{B}{(1+i)^n}$$

Here, A = 10% of 1000 = 100

r = 20%, c = 1

n = 3

B = 1000

$$\text{Thus, Valuation of bond} = \frac{100((1+0.20)^3 - 1)}{0.20(1+0.20)^3} + \frac{1000}{(1+0.20)^3}$$

$$= \frac{100((1.20)^3 - 1)}{0.20(1.20)^3} + \frac{1000}{(1.20)^3} = 210.65 + 578.70 = 789.35$$

Hence, the correct option is (b).

PRACTICE QUESTIONS (PART I)

1. An investor is considering purchasing a four-year bond with a face value of ₹10,000 and a nominal interest rate of 6%. The investor requires a rate of return of 8%. What is the maximum price the investor should be willing to pay for the bond?
   (a) ₹9000 (b) ₹9500 (c) ₹10000 (d) ₹10500
2. An investor intends purchasing a three year ₹1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?
   (a) ₹1000 (b) ₹887.152 (c) ₹907.125 (d) None of these

Answer Key

1. (b) 2. (c)

PERPETUITY

When you get an annuity for unlimited time, it becomes Perpetuity.

We need to understand two basic things:

1. When we talk about money far in the future, the present value of it becomes today extremely low.
2. And principal is never repaid, that means we do not have to think about the principal.

Then $\ddot{u}_{\infty} = \frac{R}{i}$

where, R = the payment or receipt each period

i = the interest rate

Example 44. Hari wants to retire and receive ₹15,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 7% compounded annually. How much will he need to set aside to achieve his perpetuity goal?

(a) ₹25,71,428 (b) ₹2,14,285 (c) ₹1,80,000 (d) None of these

Sol. (a) Given, R = ₹15,000, c = 12 and r = 7%

$$\Rightarrow i = \frac{\ddot{u}}{12} = \frac{}{1200}$$

We know that,

$$PVA_{\infty} = \frac{R}{i} = \frac{15000}{\frac{7}{1200}} = \frac{15000}{7} \times 1200 = ₹25,71,428 \text{ (approx.)}$$

Therefore, Hari need ₹25,71,428 to set aside to achieve his perpetuity goal.

Hence, the correct option is (a).

CALCULATING GROWING PERPETUITY

Growing perpetuity means the periodic instalment is increasing with fixed interest rate

$$PVA = \frac{A}{i-g}$$

Example 45. Assuming that the discount rate is 8% per annum, how much would you pay to receive ₹1000, growing at 4%, annually, forever?

(a) ₹25,000 (b) ₹2,500 (c) ₹1,00,000 (d) None of these

Sol. (a) Given, A = ₹1000

r = 8%

$$g = 4\% = \frac{4}{100} = 0.04$$

Here, c = 1

$$\Rightarrow i = 8\% = \frac{8}{100} = 0.08$$

Therefore, $PVA = \frac{A}{i-g}$

$$= \frac{1000}{0.08-0.04} = \frac{1000}{0.04} = 25,000$$

Therefore, the correct option is (a).

PRACTICE QUESTIONS (PART J)

1. Calculate the present value of a perpetual payment stream of ₹30,000 per month at an annual interest rate of 15%.

(a) ₹23,60,000 (b) ₹30,00,000 (c) ₹24,00,000 (d) ₹10,80,000

2. A person wants to save for his retirement and plans to make annual deposits of ₹50,000 at the end of each year for perpetuity. If the rate of interest is 12%. What is the present value of his retirement fund?
(a) ₹500,000 (b) ₹416,667 (c) ₹600,000 (d) ₹450,000

3. Ramesh wants to retire and receive ₹3,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 8% compounded annually. How much will he need to set aside to achieve his perpetuity goal?
(a) ₹4,00,000 (b) ₹4,49,775 (c) ₹5,59,775.25 (d) None of these

4. If the discount rate is 16% per annum, then how much a company has to pay to receive ₹360 growing at 10% annually forever?
(a) ₹6,000 (b) ₹5,800 (c) ₹5,400 (d) ₹5,200

5. At a discount rate of 8% per year, what would be the present value of receiving ₹300 growing at an annual rate of 6% indefinitely?
(a) ₹11,500 (b) ₹12,000 (c) ₹15,000 (d) ₹10,000

Answer Key
1. (c) **2.** (b) **3.** (b) **4.** (a) **5.** (c)

RATE OF RETURN

Net Present Value Technique (NPV)
Net present value = Present value of cash inflow – Present value of cash outflow

Example 46. Compute the net present value for a project with a net investment of ₹1,00,000 and net cash flows year one is ₹55,000 for year two is ₹80,000 and for year three is ₹15,000. Further, the company's cost of capital is 10%.
(a) ₹50,005 (b) ₹49,995 (c) ₹27,340 (d) ₹1,27,340

Sol. (c) As per the definition,
Net present value = Present value of cash inflow – Present value of cash outflow
Given, Present value of cash outflow = ₹1,00,000

Year	Net Cash Flows	PVIF @ 10%	Discounted Cash Flows
0	(1,00,000)	1.000	(1,00,000)
1	55,000	0.909	49,995
2	80,000	0.826	66,080
3	15,000	0.751	11,265
Net Present Value			27,340

Therefore, the net present value is ₹27,340.
Hence, the correct option is (c).

DECISION RULE

If NPV > 0, Accept the Proposal
If NPV < 0, Reject the Proposal

COMPOUND ANNUAL GROWTH RATE (CAGR)

It is the method to find the smoothed annualized gain of an investment over a given time period.

$$CAGR(t_o, t_n) = \left(\frac{V(t_n)}{V(t_o)}\right)^{\frac{1}{t_n - t_o}} - 1$$

Where $V(t_o)$ = Beginning Period; $V(t_n)$ = End Period

Trick:

Steps to find n^{th} root of any number:

1. Write the number.
2. Press √ ..... 12 times
3. −1
4. ÷n
5. +1
6. ×= .... 12 times

Example 47. A company recorded its annual revenues over a five-year period as follows:

Year	2001	2002	2003	2004	2005
Revenue (in thousand ₹)	100	110	121	133	146.41

What is the compound annual growth rate (CAGR) of the company's revenue over this five-year period?

(a) 10% (b) 20% (c) 30% (d) 40%

Sol. (a) We know that,

$$CAGR(t_o, t_n) = \left(\frac{V(t_n)}{V(t_o)}\right)^{\frac{1}{t_n - t_o}} - 1$$

where $V(t_o)$ is beginning period and $V(t_n)$ is end period

According to the question,

Beginning Value, $V(t_o)$ = ₹100,000

Ending Value, $V(t_n)$ = ₹146,410

Here, $t_n - t_o = 2005 - 2001 = 4$

∴ CAGR for year 5 with respect to year 1 is given by:

$$\left(\left(\frac{146410}{100000}\right)^{\frac{1}{4}} - 1\right) \times 100\%$$

$= (1.1 - 1) \times 100\% = 0.1 \times 100\% = 10\ \%$

Calculation trick:
To find 4th root
1. 146410 ÷ 100000
2. Press root 12 times
3. −1
4. ÷4 (because of fourth root)
5. +1
6. x= ..... 12 times

Therefore, the required compound annual growth rate is 10%.

Hence, the correct option is (a).

NOMINAL RATE OF INTEREST

Nominal Interest Rate = Real Interest Rate + Inflation

Example 48. If the nominal rate of growth is 17% and inflation is 9% for five years. Let P be the Gross Domestic Product (GDP) amount at the present year then the projected real GDP after 6 years is

(a) 1.587 P (b) 1.921 P (c) 1.403 P (d) 2.51 P

Sol. (a) Given, Nominal rate of return = 17% p.a.

Inflation = 9% p.a.

Effective Interest = 17% − 9% = 8% p.a.

$= \frac{8}{100} = 0.08$ p.a.

Let P be the Gross Domestic Product i.e. Present value,

Future Value is given by the formula,

$F.V = P.V(1 + i)^n = P(1 + 0.08)^6 = P\,(1.5868) \approx 1.587\,P$

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART K)

1. If P = ₹12,000, T = 3 years and I = ₹1,800, then R will be

(a) 2.5% (b) 5% (c) 7.5% (d) 10%

2. If the discount rate is 14% per annum, then how much a company has to pay to receive ₹280 growing at 9% annually forever?

(a) ₹5600 (b) ₹2,800 (c) ₹1,400 (d) ₹4,200

3. Let the operating profit of a manufacturer for five years is given as:

Year	1	2	3	4	5	6
Operating profit (in lakh ₹)	90	100	106.4	107.14	120.24	157.35

Then operating profit of compound annual growth rate (CAGR) for year 6 with respect to year 2 is given at

(a) 9% (b) 12% (c) 11% (d) 13%

4. The profit (in lakh ₹) of a company for the past five years are as follows:

Year	1	2	3	4	5
Profit	20	25	30	35	40

Find the compound annual growth rate (CAGR) of the profits for the period from year 1 to year 5.

(a) 15% (b) 19% (c) 25% (d) 35%

Answer Key

1. (b) **2.** (a) **3.** (b) **4.** (b)

PRACTICE QUESTIONS (PART I)

1. If P = ₹5,000 R = 15%, T = T = 4 $\frac{1}{2}$ years, using I = $\frac{PTR}{100}$, I will be

(a) ₹3,375 (b) ₹3,300 (c) ₹3,735 (d) ₹3,300

2. A sum was invested for 3 years as per C.I. and the rate of interest for first year is 9%, 2nd year is 6% and 3rd year is 3% p.a. respectively. Find the sum if the amount in the three years is ₹550?

(a) ₹250 (b) ₹300 (c) ₹462.16 (d) ₹350

3. A sum of money amount to ₹6,200 in 2 years and ₹7400 in 3 years, then the principal is

(a) ₹3000 (b) ₹3,500 (c) ₹3800 (d) none

4. The certain sum of money became ₹692 in 2 years and ₹800 in 5 years, then the principal amount is

(a) ₹520 (b) ₹620 (c) ₹720 (d) ₹820

5. How much will ₹25000 amount to in 2 years at compound interest if the rates for the successive years are 4% and 5% per year?

(a) ₹27300 (b) ₹27000 (c) ₹27500 (d) ₹27900

6. Assuming that the discount rate is 8% per annum, how much would an investor be willing to pay to receive ₹500 growing at a rate of 4% annually forever?

(a) ₹12,500 (b) ₹15,000 (c) ₹10,000 (d) ₹8,000

7. If the difference between the compound interest compounded annually and simple interest on a certain amount at 10% per annum for two years is ₹372, then the principal amount is

(a) ₹37200 (b) ₹37000 (c) ₹37500 (d) None of these

8. A certain sum of money Q was deposited for 5 year and 4 months at 4.5% simple interest and amounted to ₹248, then the value of Q is

(a) ₹200 (b) ₹210 (c) ₹220 (d) ₹240

9. A man deposited ₹8000 in a bank for 3 years at 5% per annum compound interest, after 3 years he will get

(a) ₹8800 (b) ₹9261 (c) ₹9,200 (d) ₹9000

10. A certain money doubles itself in 10 years when deposited on simple interest. It would triple itself in

(a) 20 years (b) 15 years (c) 25 years (d) 30 years

11. If ₹10,000 is invested at 8% per year compounded quarterly, then the value of the investment after 2 years is **(Given: $(1 + 0.2)^8 = 1.171659$)**

(a) ₹11, 716.59 (b) ₹10,716.59 (c) ₹117.16 59 (d) None of these

12. A bank pays 10% rate of interest, interest being calculated half yearly. A sum of ₹400 is deposited in the bank. The amount at the end of 1 year will be

(a) ₹440 (b) ₹439 (c) ₹441 (d) ₹442

13. A person borrows ₹5000 for 2 years at 4% per annual simple interest. He immediately lends to person at $6\frac{1}{4}$% per annum for 2 years, find his gain in the transaction per year.

(a) ₹112.50 (b) ₹225 (c) ₹125 (d) ₹107.50

14. Mr. X invests ₹10000 every year starting from today for the next 10 years. Suppose interest rate is 8 % per year compounded annually. Calculate future value of the annuity.

(a) ₹1,56,454.88 (b) ₹1,56,554.88 (c) ₹1,44,865.625 (d) None of these

15. If ₹1000 be invested at interest rate of 5% and the interest be added to the principal every 10 year, than the number of years in which it will amount to ₹2000 is:

(a) $16\frac{2}{3}$ years (b) $6\frac{1}{2}$ years (c) 16 years (d) $6\frac{2}{3}$ years

16. If compounded interest on any sum at the rate of 5% for two years is ₹512.50, then the sum would be:

(a) ₹3000 (b) ₹4000 (c) ₹5000 (d) ₹6000

17. In how many years will the sum of money become four times at 12% p.a. simple interest?

(a) 18 years (b) 21 years (c) 25 years (d) 28 years

18. How much amount is required to be invested every year to accumulate ₹7,96,870 at the end of 10 years, if interest is compounded annually at 10%?

(a) ₹40,000 (b) ₹45,000 (c) ₹48,000 (d) ₹50,000

19. A person lends ₹6000 for 4 years and ₹8000 for 3 years at simple interest. If he gets ₹2,400 as total interest, then the rate of interest is:

(a) 5% (b) 4% (c) 6% (d) 7%

20. If a person bought a house by paying ₹45,00,000 down payment and ₹80,000 at the end of each year till the perpetuity, assuming the rate of interest as 16%, the present value of house (in ₹) is given as

(a) ₹47,00,000 (b) ₹45,00,000 (c) ₹57,80,000 (d) ₹50,00,000

21. The future value of annuity of ₹2,000 for 5 years at 5% compounded annually is given (in nearest ₹) as

(a) ₹51,051 (b) ₹21,021 (c) ₹11,051 (d) ₹61,254

22. Assuming that the discount rate is 7% p.a. How much would they pay to receive ₹200 growing at 5% annually forever?

(a) ₹2,500 (b) ₹5,000 (c) ₹7,500 (d) ₹10,000

23. ₹800 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annually after 10th payment?

(a) ₹4,444 (b) ₹8,756 (c) ₹3,491 (d) ₹8,182

24. Rajan is planning for his retirement and desires to receive a monthly payment of ₹7,500. He wishes to ensure that this monthly payment continues to be passed on to future generations even after his demise. With an expected interest rate of 5% compounded annually, how much does Rahul need to set aside to achieve his perpetuity goal?

(a) ₹21,00,000 (b) ₹15,00,000

(c) ₹24,00,000 (d) ₹18,00,000

25. A person wants to lease out a machine costing ₹5,00,000 for a 10 year period. It has fixed a rental of ₹51,272 per annum payable annually starting from the end of first year. Suppose rate of interest is 10% per annum, compounded annually on which money can be invested. To whom this agreement is favourable?

(a) Favour for lessee (b) Favour for lessor

(c) Not for both (d) Can't be determined

Answer Key

1. (a)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (b)	**10.** (a)
11. (a)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (d)	**19.** (a)	**20.** (d)
21. (c)	**22.** (d)	**23.** (d)	**24.** (d)	**25.** (a)					

SUMMARY

- The sum of money received in the future is less valuable than it is today.
- **Interest:** Price paid by the borrower for the use of lender's money, Or we say if we borrow some money from a person for a certain time period then the extra amount paid than the initial amount is called interest.
- **Simple interest:** It is interest computed on principal for entire period of borrowing

Formula : $S.I. = \frac{P \times R \times T}{100}$

$A = P + S.I.$

where,

A = Final value of an investment (accumulated amount)

P = Initial value of an investment (Principal amount)

R = rate of interest in decimal

S.I. = amount of interest

T = time in years

- **Compound interest:** Interest that occurs when earning for each specified time period is added to the principal amount.

  Formula: $C.I = P\left(1 + \frac{r}{100}\right)^n - P$

  $A = P\left(1 + \frac{r}{100}\right)^n$

  where,

  P = Initial value of an investment (Principal Amount)

  r = Annual rate of interest

  n = t × no. of conversions per year (c)

- **Effective rate of interest:** Effective interest rate is calculated by formula

  $E = \left(\left(1 + \frac{r}{100c}\right)^c - 1\right) \times 100\%$

- **Annuity** can be defined as a sequence of periodic payments (or receipts) regularly over a specified period of time.
  - Annuity may be of two types:

  (i) Annuity Regular: In annuity regular first payment/receipt takes place at the end of first period.

  (ii) Annuity Due or Annuity Immediate: When the first receipt or payment is made today (at the beginning of the annuity) it is called annuity due or annuity immediate.

- **Future Value of Annuity Regular:** If A be the periodic payments, the future value A (n,i) of the annuity is given by

  $A(n,i) = A\left[\frac{(1+i)^n - 1}{i}\right]$

- **Future value of an Annuity Due/Annuity Immediate** = Future value of annuity regular × (1 + i) where i is the interest rate in decimal

- **Present value of an Annuity Regular:** The present value P of the Annuity A due at the end of n period at the rate of r per interest period may be obtained by

  $P.V. = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$

- **Present Value of an Annuity Due or Annuity Immediate:**

  (i) Calculate the present value for (n – 1) period as annuity regular.

  (ii) Add the initial payment (receipt) in the above.

- **Capital Expenditure (investment decision):** Capital expenditure means purchasing an asset (which results in outflows of money) today in anticipation of benefits (cash inflow) which would flow across the life of the investment.

- **Valuation of Bond:** A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issued for a fixed term longer than one year.

- **Perpetuity:** When you get an annuity for unlimited time, it becomes Perpetuity.

$$PVA_{\infty} = \frac{R}{i}$$

- **Calculating Growing Perpetuity:**
  Growing perpetuity means the periodic instalment is increasing with fixed interest rate.

$$\ddot{u} \quad = \frac{A}{i-g}$$

- **Rate of Return:**
  Net present value = Present value of cash inflow – Present value of cash outflow
- **Compound Annual Growth Rate (CAGR):**
  It is the method to find the smoothed annualized gain of an investment over a given time period

$$CAGR(t_o, t_n) = \left(\frac{V(t_n)}{V(t_o)}\right)^{\frac{1}{t_n - t_o}} - 1$$

where, V (t_o) = Beginning Period and V (t_n) = End Period

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